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Self and Mutual Admittances of Waveguides Radiating Into Plasma Layers

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The self and mutual admittance of waveguide backed slots radiating into a plasma (or dielectric) layer has been formulated in a laterally unbounded geometry. The admittance expression involves integrals that may be approximated by summations for numerical calculations. For a constant mesh size approximation to the integrals the summations are the same as obtained in earlier work, where the plasma layers are considered to be within a wide waveguide. The accuracy of the solution is improved by decreasing the mesh size used for the numerical integration, which is equivalent to increasing the size of the large guide in the waveguide model.

Mutual admittance calculations between two-parallel slots show that the presence of a plasma layer decreases the mutual admittance relative to its free space value.

1. Introduction

The problem of a plasma covered antenna has been treated in numerous papers. Line-source excited plasma layers have been considered by Newstein and Lurye [1956]; Yee [1961]; Tamir and Oliner [1962]; Shore and Meltz [1962]; and Hodara and Cohn [1962]. Three-dimensional treatments of plasma (or dielectric) covered antennas have been carried out by Katzin [1957]; Marini et al. [1958]; Marini [1960]; Wait [1959]; [1961]; Knop [1961]; Hodara [1963], and by Knop and Cohn [1964]. However, this work does not consider the effects of the plasma layer on the field distribution along the antenna. An iteration technique is used by Raemer [1962] in considering the effects of a plasma shell on the current distribution along the antenna, but there are no satisfactory results for the antenna impedance. A variational formulation of a slot impedance has been worked out by Cutler [1959]. The fields in the plasma layer and in the outside space are considered as a superposition of waveguide modes. There appear to be computational difficulties and this formulation has not yielded workable equations or any numerical results.

Another variational formulation for the slot impedance has been given by Galejs [1964b]. The dyadic Green's function for a plasma layer of finite thickness is obtained from the free space Green's function by the method of images. The fields outside the slot depend on a surface integral of the fields over the slot plane and over the surface of the plasma layer. For plasma layers of a thickness greater than a wavelength the surface fields are related to the fields in the slot plane with the aid of plane wave reflection coefficients, which leads to an integral equation for the fields in the slot plane. This formulation [Galejs, 1964b] is rather involved and it does not apply to thin or stratified plasma layers.

Cohn and Flesher [1958] have discussed radiation from a coaxial line into a waveguide the walls of which are removed to infinity, and their results are in agreement with free space analyses [Levine, and Papas, 1951]. The waveguide boundary greatly simplifies the admittance calculations for a plasma or dielectric covered slot, if it is permissible to use a waveguide of finite diameter. Such calculations have been carried out for an annular slot within a large circular waveguide by Galejs [1964a]. The dimensions of the large guide are selected in such a way that none of the waveguide modes of the free space portion of this guide are near their respective cutoffs. The admittance of the slot which radiates into the larger waveguide now approximates the slot admittance for a laterally unbounded plasma layer.

A similar formulation has been used for rectangular slots which are assumed to radiate into a wide rectangular waveguide [Galejs, 1965]. Calculations have been made for a superposition of sine and cosine functions or for the principal mode electric or magnetic field as trial functions for aperture fields.

The slot admittance of an open waveguide radiating into a laterally unbounded plasma has

been recently calculated by Villeneuve [1965]. He assumes the principal waveguide mode for the aperture field, and the slot admittance which is obtained after numerically evaluating a two-fold infinite integral corresponds closely to the admittance computed in the waveguide formulation by Galejs [1965].

Although the numerical results obtained from the waveguide geometry are reasonably close to results available for a laterally unbounded geometry [Galejs, 1964b; Villeneuve, 1965], there have been no strict justifications for the use of the waveguide model [Galejs, 1964a, 1965]. However, such a justification can be provided by deriving the slot admittance first for a laterally unbounded geometry, and by examining the conditions under which this is approximated by the admittance of the waveguide model. It will be shown that the admittance of the waveguide model may be obtained after approximating the infinite integrals of the rigorous admittance expression by discrete summations. Thus, the waveguide model is equivalent to a particular constant mesh approximation of the integrals in a rigorous admittance expression, but a constant mesh approximation is not necessarily the best for accurate numerical results, as will be shown in section 2.

In past work [Yee, 1961] the mutual admittance has been computed only for parallel line sources and thick plasma layers. The present formulations can be adapted for mutual impedance calculations between waveguide backed slots for isotropic plasma (or dielectric) layers of arbitrary parameters. This work is discussed in section 3.

In this paper the plasma is represented as an isotropic lossy dielectric having a relative dielectric constant less than unity. Also, the perturbations of the plasma properties by the antenna fields are ignored. The above representation is valid only in low power applications.

2. Self Admittance

2.1. Rectangular Waveguide

The waveguide geometry under consideration is depicted in figure 1. A rectangular waveguide of cross-sectional area $x_i y_i$ is joined by a symmetrical aperture of width 2ϵ and length $2l$ to an infinite flange. Layered isotropic plasma (or dielectric) fills the space $0 \leq z \leq z_1$, and there is free space for $z > z_1$. The waveguide is excited in the principal (TE_{10}) mode, but the change of the waveguide cross section and the dielectric discontinuities generate a large number of TE and TM modes which must be considered in the formulation of the slot admittance. Such calculations have been carried out in the appendix and the following stationary expression for the normalized slot admittance y_s is derived:

$$y_s=\frac{x_i y_i}{\gamma_{i1} D^2}\left\{\sum_n \sum_m{}' \frac{\epsilon_m}{x_i y_i \gamma_i}\left[(\beta_{xi}^2-k_i^2)\left(\iint E_{yi}\right)^2+(k_i^2-\beta_{yi}^2)\left(\iint E_{xi}\right)^2\right]+I\right\} \quad (1)$$

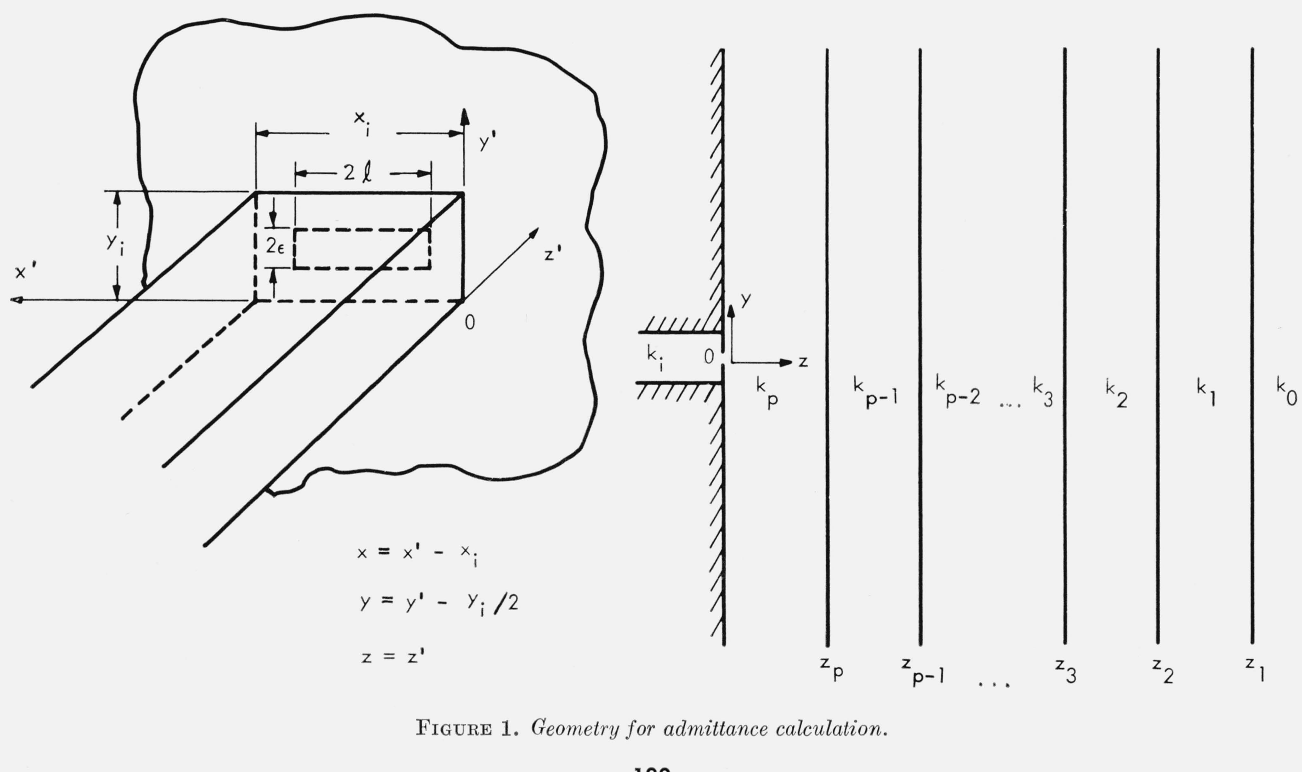

FIGURE 1. *Geometry for admittance calculation.*

where

$$I=\frac{1}{8\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[F(R)\left(\iint E_{yp}\right)^2 + G(R)\left(\iint E_{xp}\right)^2\right]dudv \quad (2)$$

$$F(R)=\frac{1}{(u^2+v^2)\gamma_p}\left[\gamma_p^2u^2\frac{1-R_a}{1+R_a}-k_p^2v^2\frac{1+R_b}{1-R_b}\right] \quad (3)$$

$$G(R)=\frac{1}{(u^2+v^2)\gamma_p}\left[-\gamma_p^2v^2\frac{1-R_a}{1+R_a}+k_p^2u^2\frac{1+R_b}{1-R_b}\right] \quad (4)$$

$$D=\iint E_y(x',y')\sin\beta_{xi1}x'dx'dy' \quad (5)$$

and

$$\iint E_{xi}=\iint E_x\cos\beta_{xi}x'\sin\beta_{yi}y'dx'dy' \quad (6)$$

$$\iint E_{yi}=\iint E_y\sin\beta_{xi}x'\cos\beta_{yi}y'dx'dy' \quad (7)$$

$$\iint E_{jp}=\iint E_je^{iux}e^{ivy}dxdy. \quad (8)$$

E_x and E_y refer to the aperture fields at $z=0$ and $j=x$ or y. The other symbols are defined as

$$\beta_{xi}=\frac{n\pi}{x_i},\ \beta_{xi1}=\frac{\pi}{x_i},\ \beta_{yi}=\frac{m\pi}{y_i},\ k_j=\sqrt{\omega^2\mu_0\epsilon_j+i\omega\mu_0\sigma_j}$$

with $j=i$ or p, $\gamma_i=i\sqrt{k_i^2-\beta_{xi}^2-\beta_{yi}^2}$, $\gamma_{i1}=i\sqrt{k_i^2-\beta_{xi1}^2}$, $\gamma_p=i\sqrt{k_p^2-u^2-v^2}$ and $\epsilon_m=1$ for $m=0$, $\epsilon_m=2$ for $m\neq 0$. The prime on the double summation (1) designates the omission of its $n=1$, $m=0$ term. The reflection coefficients of the TE modes R_a and of the TM modes R_b depend on the dielectric structure for $z>0$. For stratified dielectric layers shown in figure 1 the reflection coefficient R_{dj} ($d=a$ or b) in the region of $z_{j+1}\leq z\leq z_j$ is related to the reflection coefficient $R_{d(j-1)}$ of the region $z_j\leq z\leq z_{j-1}$ by the expressions

$$R_{aj}=e^{2\gamma_jz_j}\frac{[e^{2\gamma_{j-1}z_j}+R_{a(j-1)}]-\frac{\gamma_{j-1}}{\gamma_j}[e^{2\gamma_{j-1}z_j}-R_{a(j-1)}]}{[e^{2\gamma_{j-1}z_j}+R_{a(j-1)}]+\frac{\gamma_{j-1}}{\gamma_j}[e^{2\gamma_{j-1}z_j}-R_{a(j-1)}]} \quad (9)$$

$$R_{bj}=e^{2\gamma_jz_j}\times\frac{\left(\frac{k_{j-1}}{k_j}\right)^2[e^{2\gamma_{j-1}z_j}+R_{b(j-1)}]-\frac{\gamma_{j-1}}{\gamma_j}[e^{2\gamma_{j-1}z_j}-R_{b(j-1)}]}{\left(\frac{k_{j-1}}{k_j}\right)^2[e^{2\gamma_{j-1}z_j}+R_{b(j-1)}]+\frac{\gamma_{j-1}}{\gamma_j}[e^{2\gamma_{j-1}z_j}-R_{b(j-1)}]} \quad (10)$$

R_{dj} is derived by considering the scalar functions (28) and (29) in two adjacent layers and by requiring the tangential field components (30), (31), (33), and (34) to be continuous across the interface. The computations should start with $j=1$ where $R_{d(j-1)}=R_{d0}=0$ in (9) and (10). A series of computations gives then R_{d1}, R_{d2}, R_{d3} and finally R_{dp} as R_a or R_b in (3) and (4).

After denoting the function within the square bracket of (2) by $H(u,v)$ I is rewritten as

$$I=\frac{1}{8\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}H(u,v)dudv. \quad (11)$$

If E_y is assumed to be an even function with respect to the x and y axis, it follows from (26), (27), (30), and (31) that E_x is an odd function. The integral (8) is then changed to

$$\iint E_{xp}=-\iint E_x\sin ux\sin vy\,dxdy \quad (12)$$

$$\iint E_{yp}=\iint E_y\cos ux\cos vy\,dxdy. \quad (13)$$

The integrals $\iint E_{xp}$ and $\iint E_{yp}$ are odd and even with respect to u or v, but their squares are both even functions, $F(R)$ and $G(R)$ are even functions and so is $H(u,v)$ with respect to u and v. After changing the variable of integration in (11) to n and m, where $u=n\pi/x_0$ and $v=m\pi/y_0$, it follows that

$$I=\frac{1}{2\pi^2}\int_0^{\infty}\int_0^{\infty}H(u,v)dudv =\frac{1}{2x_0y_0}\int_0^{\infty}\int_0^{\infty}H(n,m)dndm. \quad (14)$$

For x_0 and y_0 sufficiently large $H(n,m)$ will change only slightly even for integral changes in n or m, and the integral may be broken down in a summation of terms

$$\int_{n_0-1}^{n_0+1}\int_{m_0-1}^{m_0+1}H(n,m)dndm\approx 4H(n_0,m_0) \quad (15)$$

for nonnegative lower limits of the integrals. The factor 4 is replaced by 2 for n_0 or $m_0=0$. Restricting n to odd integers and m to even integers, I is approximated by the summation

$$I\approx\frac{1}{x_0y_0}\sum_n\sum_m\epsilon_mH(n,m). \quad (16)$$

Using this restriction of permissible n and m values, (1) and (16) can be seen to agree with corresponding expressions of Galejs [1965] if the differences in the definition of the x and y coordinates of the two papers are considered. Hence the slot admittance, computed from (1) and (16) is the same as for the waveguide model of Galejs [1965]. The parameters x_0 and y_0 are now identified as the dimensions of a large waveguide, which encloses the plasma layer laterally.

In the special case of wide open waveguide ($x_i=2l$, $y_i=2\epsilon$) and for the principal waveguide mode as the aperture field E_y with $E_x=0$, the primed double summation of (1) is equal to zero. Also $\iint E_{xp}=0$

and $\int\int E_{yp}$ of (13) is given by

$$\int\int E_{yp} = -\frac{4\beta_{xi1} E_y(0, y) \cos ul \sin v\epsilon}{(u^2 - \beta_{xi1}^2) v}. \tag{17}$$

After changing to the variables n and m, (17) is seen to give the same slot admittance (1) as (26) and (27) of Galejs [1965].

The ability of the waveguide model to approximate a laterally unbounded geometry is dependent on the accuracy of (15). For large values of x_0 and y_0 the increments of $H(n, m)$ between successive values of n and m are decreased and the approximation (15) is more accurate. However, the accuracy is not improved monotonically by increasing x_0 and y_0. There is a possibility that $H(n_0, m_0)$ approaches infinity while the integral in the left-hand side of (15) remains finite. This may occur near the cutoff points of waveguide modes as was pointed out by Galejs [1965]. Such values of x_0 and y_0 should be avoided, or the accuracy of the approximation (15) should be improved by numerical integration techniques in the vicinity of such points. However, a smaller accuracy can be tolerated for large arguments of the integrals, that represent attenuated waves.

Equation (15) represents only a particular approximation and different summations may be obtained for a waveguide with magnetically conducting walls or by using a variable mesh size in the process of approximating the double integral (14). Using the waveguide analogy this corresponds to values of x_0 and y_0 which depend on mode numbers n and m. The dimensions of an equivalent waveguide are therefore changed for different waveguide modes.

2.2. Circular Waveguide

A similar interpretation applies also to plasma layers in a cylindrical waveguide model. For a laterally unbounded plasma layer, the admittance of an annular slot is given as in (17) of Galejs [1964a], but the n-summation is changed to an integral

$$I = \frac{1}{2} \int_0^\infty \frac{\sigma_2 + j\omega\epsilon_2}{\gamma_2} \frac{1+B_2}{1-B_2} \left[\frac{J_0(\lambda\rho_1) - J_0(\lambda\rho_2)}{\lambda}\right]^2 \lambda d\lambda$$

$$= \frac{1}{2} \int_0^\infty H(\lambda)\lambda d\lambda \tag{18}$$

where γ_2 and B_2 are functions of a continuous variable λ as shown for discrete values of λ_n in (8) and (19) of Galejs [1964a]. The integral may be approximated by a summation of terms

$$I_n = \tfrac{1}{2} \int_{\lambda_n - \Delta_1}^{\lambda_n + \Delta_2} H(\lambda)\lambda d\lambda \approx \frac{1}{2} H(\lambda_n)\lambda_n(\Delta_1 + \Delta_2) \tag{19}$$

where $\Delta_1 + \Delta_2 = \frac{1}{2}(\lambda_{n+1} - \lambda_{n-1})$. The zeros of the Bessel function

$$J_0(\lambda_n c) = 0 \tag{20}$$

may be chosen for the sequence of λ_n's. In the first approximation $(\lambda_n c) \approx (n - \frac{1}{4})\pi$ and

$$I_n \approx \tfrac{1}{2} H(\lambda_n)\pi^2(n - \tfrac{1}{4})/c^2 \approx H(\lambda_n)/[cJ_1(\lambda_n c)]^2 \tag{21}$$

with an error of less than 2 percent for $n \geq 1$. The I_n of (21) corresponds to the individual terms of the n-summation in (17) of Galejs [1964a].

The admittance, which is computed for laterally unbounded plasma layers with the aid of (18), can be approximated by a sum of waveguide modes for a waveguide of radius c as shown in (17) of Galejs [1964a], provided that the approximation (19) is valid. Selection of small values of Δ_1 and Δ_2 or of large values of c will improve its accuracy, if singular values of $H(\lambda_n)$ are avoided in the right-hand side of (19).

3. Mutual Admittance

The mutual admittance between two rectangular waveguides radiating into a plasma layer is computed in the geometry of figure 2. The plasma layers which are above the $z=0$ plane are not shown explicitly. The mutual admittance can be determined from the reaction theorem, which has been generalized by Richmond [1961] to apply to two different antenna environments, such as indicated in figure 3. First, antenna No. 1 is excited with the waveguide opening No. 2 short circuited. Second, antenna No. 2 is excited and the waveguide opening No. 1 is short circuited. The electric field excited by antenna 1 is normal to the ground plane over the aperture of antenna 2 and (7) of Richmond [1961] is simplified to

$$\sum_n I_{21n} V'_{22n} = \int_{S_2} \mathbf{E}'_2 x \mathbf{H}_1 \cdot d\mathbf{s} \tag{22}$$

where I_{21n} is the current amplitude of waveguide mode n at 2 due to the excitation at 1, V'_{22n} is the voltage of mode n at 2 due to excitation at 2, $\mathbf{E}'_2$ and $\mathbf{H}_1$ are the transverse electric and magnetic field components at 2 due to excitation of aperture 2 and 1 respectively. The I_{21n} and V'_{22n} are computed following standard definitions of waveguides [Marcuvitz, 1951], S_2 corresponds to the cross-sectional area of the waveguide, that may be larger than the waveguide aperture, and $d\mathbf{s}$ is normal to the aperture plane. The mutual admittance in the fundamental waveguide modes is computed from (22) as

$$Y_{21} = \frac{I_{210}}{V_{110}} = \frac{1}{V_{110} V'_{220}} \left[\int_{S_2} \mathbf{E}'_2 x \mathbf{H}_1 \cdot d\mathbf{s} - \sum_{n=1}^{\infty} I_{21n} V'_{22n}\right] \tag{23}$$

where $n=0$ denotes the fundamental TE_{10} waveguide mode and $n \neq 0$ denotes the higher TE or TM modes. After relating $\mathbf{H}_1$ and I_{21n} to the tangential electric field $\mathbf{E}_1$ at the aperture 1, the mutual admittance is specified in terms of the tangential aperture fields $\mathbf{E}_1$ and $\mathbf{E}'_2$. Because of the stationary character of

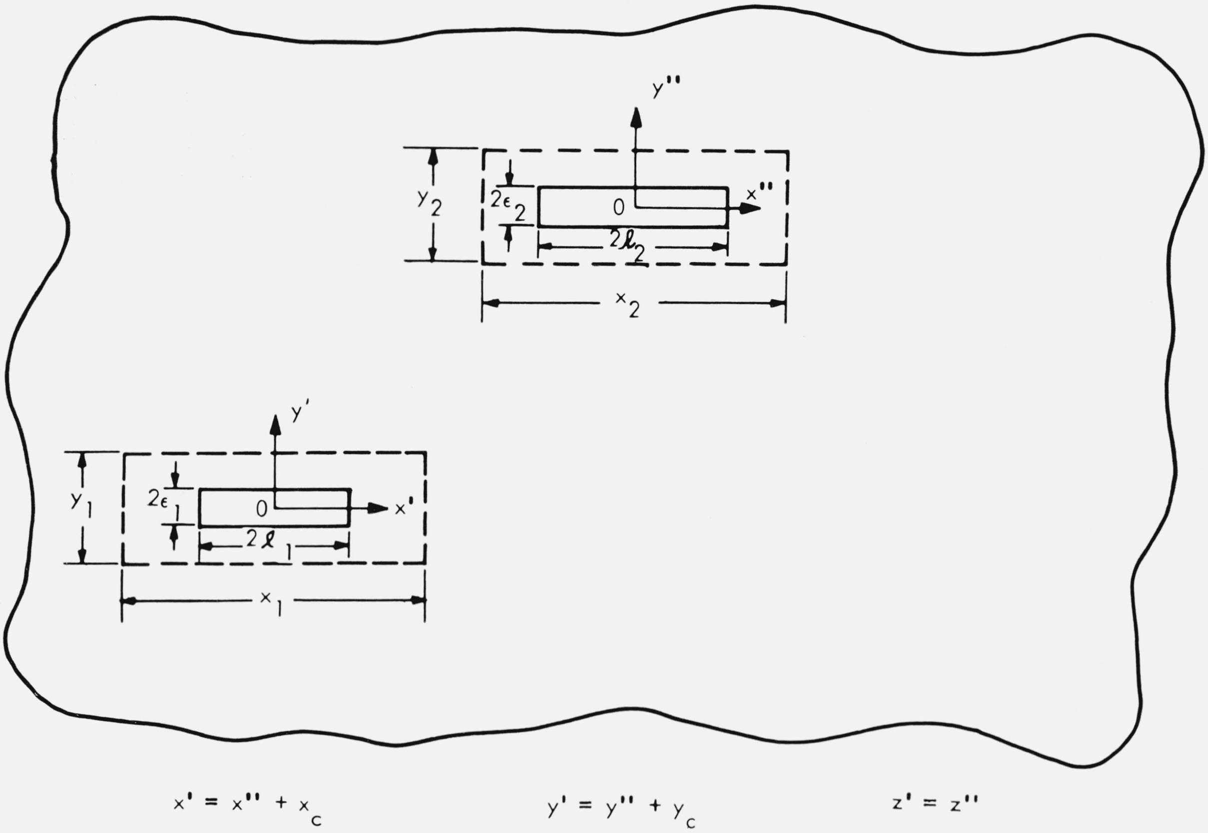

FIGURE 2. *Waveguide geometry for calculating mutual admittance.*

the reactions [Harrington, 1961, p. 341], (23) is stationary for small variations of trial fields $\mathbf{E}_1$ and $\mathbf{E}_2'$ about their correct values, and it is formally possible to determine approximations to the aperture fields $\mathbf{E}_1$ and $\mathbf{E}_2'$ similarly as has been done in self-impedance calculations [Galejs, 1965]. However, the computations are considerably simplified by assuming that the electric aperture fields may be represented in the first approximation by fields of the principal mode. In this case $\mathbf{E}_2'$ and $\mathbf{E}_1$ have only a y-component, further all the terms of the summation are equal to zero for wide open waveguides ($y_j=2\epsilon_j$, $x_j=2l_j$) and

$$Y_{21}=-\frac{1}{V_{110}V_{220}'}\int_{S_2} E_{y2}'H_{x1}dxdy. \tag{24}$$

After computing H_{x1} from (28), (29) and (33) it follows that

$$Y_{21}=\frac{1}{2\pi^2 i\omega\mu_0\sqrt{x_1y_1x_2y_2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(R)\left[\frac{\iint E_{yp1}}{E_{y1}(0,y)}\right] \times\left[\frac{\iint E_{yp2}}{E_{y2}'(0,y)}\right]e^{-i(ux_c+vy_c)}dudv \tag{25}$$

where $\int\int E_{ypj}(j=1 \text{ or } 2)$ follows from (17) by using the appropriate values of β_{xi1}, l and ϵ for the two waveguides. The admittance Y_{21} can be normalized by dividing it by the characteristic admittance of one of the waveguides, which is $Y_0=\gamma_{i1}/(i\omega\mu_0)$. For x_c and $y_c\rightarrow 0$ and $x_1=x_2$ and $y_1=y_2$ the two slots of figure 2 degenerate into a single slot and the expression for the mutual admittance (25) gives the same self admittance of a slot as the earlier calculation in (1) and (17) for the same antenna parameters.

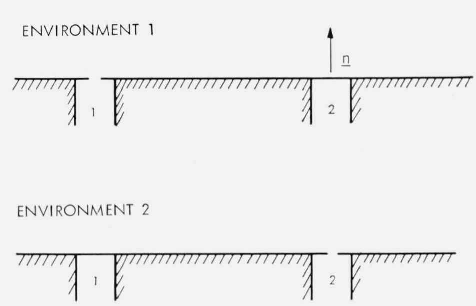

FIGURE 3. *Antenna environments for defining the mutual admittance by (22).*

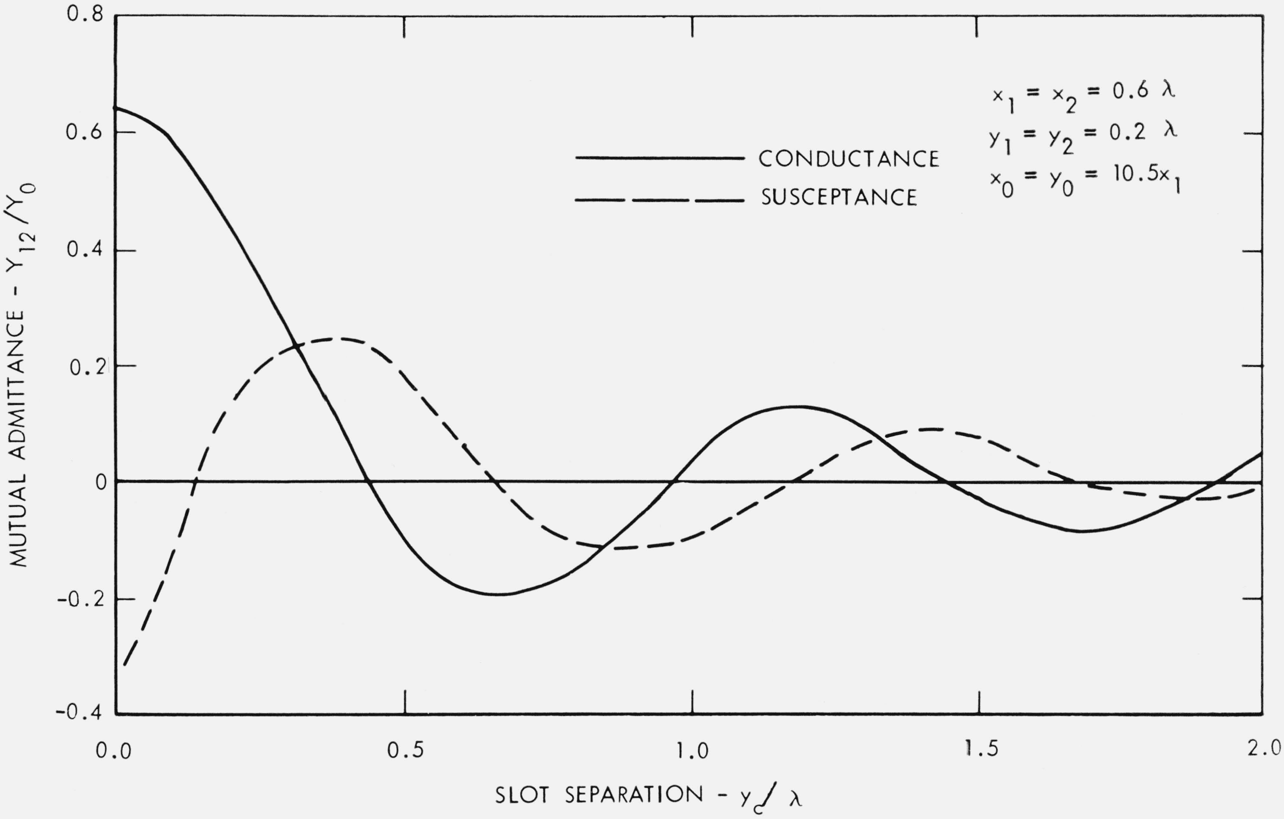

FIGURE 4. *Free space mutual admittance between two waveguide apertures.*

The double integral (25) can be approximated by a double summation as shown in (11) to (16). When changing to the discrete variables n and m where $u=n\pi/x_0$ and $v=m\pi/y_0$, x_0, and y_0 can be interpreted as the dimensions of a large waveguide, which encloses the plasma layer laterally.

Numerical calculations are made for two parallel slots ($x_c=0$) of equal size ($x_1=x_2$, $y_1=y_2$) that radiate into free space or in a single plasma layer of thickness z_p. The integrals (25) are approximated by summations with $x_0=y_0=10.5x_1$, and the accuracy of the procedure is tested by increasing $x_0=y_0$ to $19.5x_1$. This gives negligible changes in the admittance figures and the mesh size in the integrals which corresponds to $x_0=y_0=10.5x_1$ was considered adequately small.

In free space with no plasma layers in front of the two waveguides the mutual admittance curves of figure 4 show about the same relative variations as mutual impedance curves of two parallel thin half-wavelength-long dipoles in free space. This applies in particular to those values of the slot separation y_c/λ where either the mutual conductance or susceptance goes through a zero.

The effects of a single lossless plasma layer of thickness $z_p=0.1\lambda$ and λ are shown in figures 5 to 7 for relative dielectric constants $\epsilon_p/\epsilon_0=0.9$, 0.5, and 0.1. The mutual admittance variations of the thin plasma layer of $\epsilon_p/\epsilon_0=0.9$ are nearly the same as for the slots in free space. The magnitude of the mutual admittance is decreased by decreasing ϵ_p and by increasing the layer thickness. The plasma layer tends to decouple two adjacent slots and there is little resemblance between the mutual admittance curves for free space in figure 4 and for $\epsilon_p/\epsilon_0=0.1$ in figure 7.

The mutual admittance with a lossy plasma layer is illustrated in figure 8 for $\epsilon_p/\epsilon_0=0.5$ and $\tan\delta_p=1$. A comparison with figure 5 shows an increased self admittance, but a decreased mutual admittance for slot separations y_c exceeding 0.4λ. There is a further decoupling of the slots by a lossy plasma layer.

The free space mutual admittance between two thin infinitely long slots [Yee, 1961] is shown in figure 9 for a comparison with calculations using a three dimensional model shown in figure 4. The mutual admittance curves of the infinitely long slots of figure 9 decay more gradually than in figure 4. Two dimensional models will therefore indicate too high a mutual admittance figure when applied to finite length slots.

4. Conclusions

It has been shown that slot admittance for plasma layers formulated in a waveguide model [Galejs, 1964a, 1965] are equivalent to the computations for a laterally unbounded plasma geometry, provided the infinite integrals of the latter formulation are

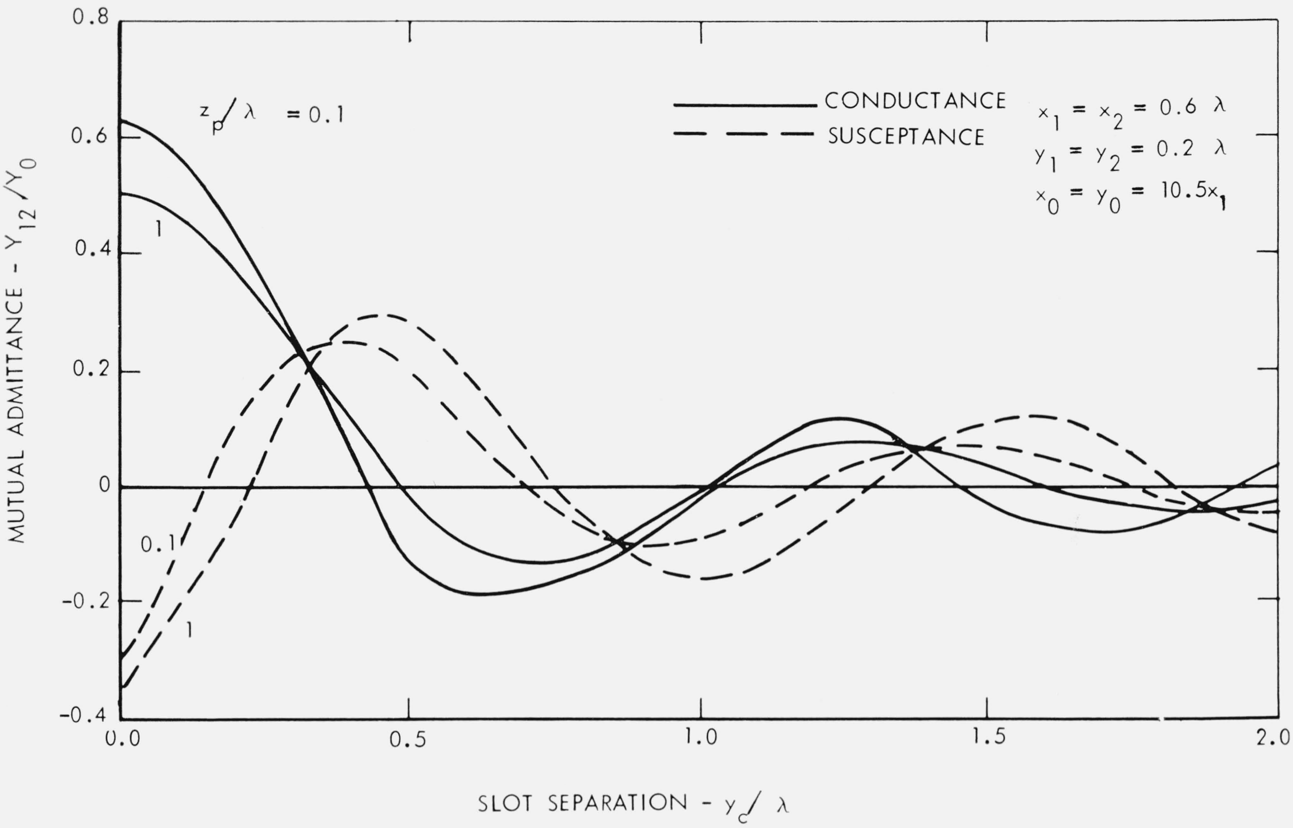

FIGURE 5. *Mutual admittance for a lossless plasma layer of* $\epsilon_p/\epsilon_0=0.9$.

approximated by summations. The accuracy of the solution is improved by decreasing the mesh size used in the numerical integrations, which is equivalent to increasing the size of the guide in the waveguide model. However, a constant mesh size approximation is not optimum, and more accurate numerical integration techniques may be advisable near singularities of the integrals. Also, a decreased accuracy of the approximations may be tolerated for large arguments of integrals that represent the attenuated modes. Still the waveguide model [Galejs, 1964a, 1965] represents a simple method for obtaining an approximate answer, the accuracy of which can be established from comparison with other work [Galejs, 1964b; Villeneuve, 1965].

A similar method has been used for computing the mutual admittance between two waveguide backed slots in the presence of a plasma layer. The mutual admittance between two slots is decreased by the presence of thick plasma layers in particular for high losses.

The formal solution presented in this paper applies also to dielectric of relative dielectric constant larger than unity, when the layer can support surface waves. The multiple reflection of trapped waves by a boundary may react back strongly on the waveguide. The input admittance will depend more critically on the parameters of the dielectric layer or on frequency changes, but no numerical calculations have been made using the waveguide model [Galejs, 1965] in such geometries.

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5. Appendix. Slot Admittance

In the geometry of figure 1 the wave guide ($z\leq 0$) is excited in its principal TE_{10} mode by waves which propagate in the positive z-direction. There are also reflected waves in this guide. Waves propagating in the positive and negative z-directions are excited in the outer space ($z\geq 0$). The fields consist of a superposition of TE and TM modes that can be derived from scalar functions Ψ_j and Φ_j respectively. The subscripts $j=i$ or p designate the guide portion of the outside space. For an assumed

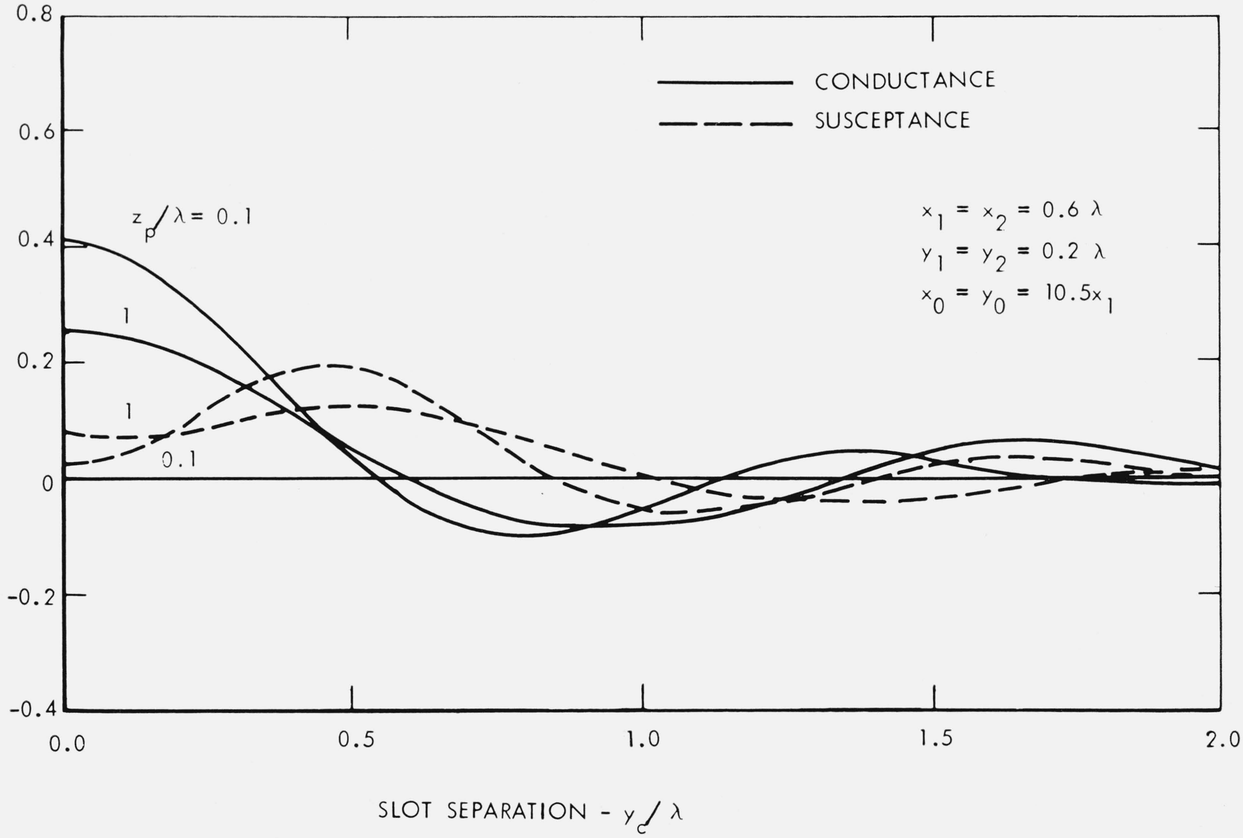

FIGURE 6. *Mutual admittance for a lossless plasma layer of* $\epsilon_p/\epsilon_0=0.5$.

exp $(-i\omega t)$ time dependence of the field it follows that

$$\Psi_i=a_{10}\cos\beta_{xi1}x'(e^{\gamma_{i1}z}+Re^{-\gamma_{i1}z})$$

$$+\sum_n\sum_m{}'\,a_{nm}\cos\beta_{xi}x'\cos\beta_{yi}y'e^{-\gamma_i z}\quad(26)$$

$$\Phi_i=\sum_n\sum_m b_{nm}\sin\beta_{xi}x'\sin\beta_{yi}y'e^{-\gamma_i z}\quad(27)$$

$$\Psi_p=\iint_{-\infty}^{\infty}A(u,v)e^{-iux}e^{-ivy}(e^{\gamma_p z}+R_a e^{-\gamma_p z})dudv\quad(28)$$

$$\Phi_p=\iint_{-\infty}^{\infty}B(u,v)e^{-iux}e^{-ivy}(e^{\gamma_p z}+R_b e^{-\gamma_p z})dudv\quad(29)$$

where the symbols are defined following (1) to (8). The field components are related to Ψ_j and Φ_j by

$$E_{xj}=\frac{\partial}{\partial y}\Psi_j+\frac{\partial^2}{\partial z\partial x}\Phi_j\quad(30)$$

$$E_{yj}=-\frac{\partial}{\partial x}\Psi_j+\frac{\partial^2}{\partial z\partial y}\Phi_j\quad(31)$$

$$E_{zj}=-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Phi_j\quad(32)$$

$$H_{xj}=\frac{1}{i\omega\mu_0}\left[\frac{\partial^2}{\partial z\partial x}\Psi_j+k_j^2\frac{\partial}{\partial y}\Phi_j\right]\quad(33)$$

$$H_{yj}=\frac{1}{i\omega\mu_0}\left[\frac{\partial^2}{\partial z\partial y}\Psi_j-k_j^2\frac{\partial}{\partial x}\Phi_j\right]\quad(34)$$

$$H_{zj}=-\frac{1}{i\omega\mu_0}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\Psi_j.\quad(35)$$

Substituting (26) and (27) in (30) and (31) the resulting expression can be solved for a_{10}, a_{nm}, and b_{nm}, which gives

$$a_{10}=\frac{2}{\beta_{xi1}x_iy_i(1+R)}\iint E_y\sin\beta_{xi1}x'dx'dy'\quad(36)$$

$$a_{nm}=-\frac{2\epsilon_m}{x_iy_i(\beta_{xi}^2+\beta_{yi}^2)}\left[\beta_{yi}\iint E_{xi}-\beta_{xi}\iint E_{yi}\right]\quad(37)$$

$$b_{nm}=-\frac{2\epsilon_m}{x_i y_i(\beta_{xi}^2+\beta_{yi}^2)\gamma_i}\left[\beta_{xi}\iint E_{xi}+\beta_{yi}\iint E_{yi}\right] \quad (38)$$

where the integrals $\int\int E_{xi}$ and $\int\int E_{yi}$ are defined by (6) and (7). Substituting (28) and (29) in (30) and (31) and solving for $A(u, v)$ and $B(u, v)$ gives

$$A(u,v)=\frac{iv\iint E_{xp}-iu\iint E_{yp}}{(2\pi)^2\gamma_p(u^2+v^2)(1+R_a)} \quad (39)$$

$$B(u,v)=\frac{iu\iint E_{xp}+iv\iint E_{yp}}{(2\pi)^2\gamma_p(u^2+v^2)(1-R_b)} \quad (40)$$

where $\int\int E_{jp}$ is defined by (8). After substituting a_{nm}, b_{nm}, $A(u, v)$ and $B(u, v)$ in (26) to (29), the magnetic field components H_{xj} and H_{yj} of (33) and (34) are related to the integrals of the electric field components (6) to (8). The tangential magnetic field components, H_x and H_y are continuous across the aperture. The H_x of (26), (27), and (33) is equal to H_x of (28), (29), and (33). Also H_y of (26), (27), and (34) is equal to H_y of (28), (29), and (34). The H_x equations are multiplied with E_y and integrated over the slot plane; the H_y equations are multiplied with E_x and also integrated over the slot plane. The resulting two equations contain terms proportional to E_x and E_y integrals.

After assuming E_y to be an even functional with respect to x and y, it follows from (30) and (31) that E_x is an odd function. It is also seen that $E_x(x, y) = E_x(-x, -y)$. Using these properties of E_y and E_x, the two earlier equations can be combined which results in (1) to (5).

The stationarity of (1) can be demonstrated by examining the first order change δy_s due to small changes δE_x and δE_y about their correct values E_x and E_y. As a result of these calculations δy_s is shown to be zero.

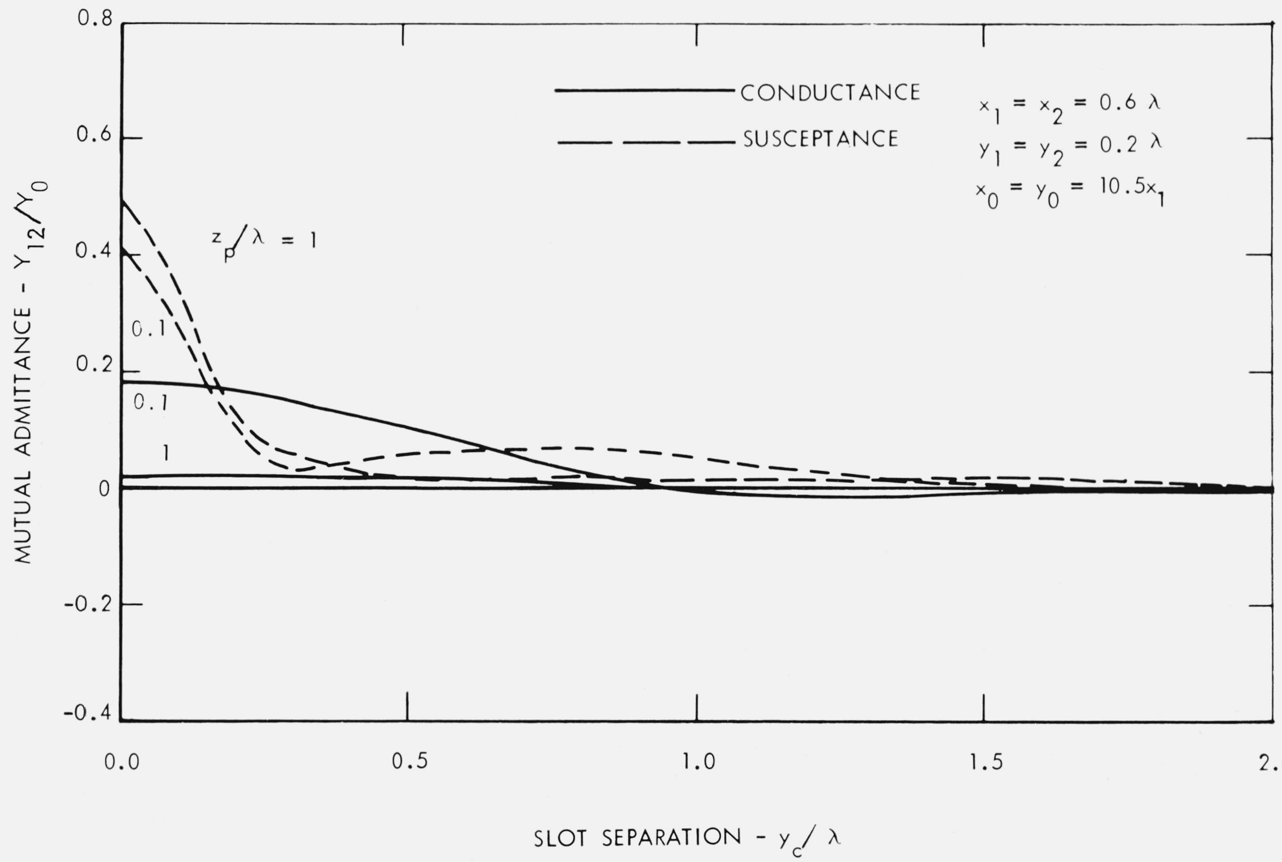

FIGURE 7. *Mutual admittance for a lossless plasma layer of $\epsilon_p/\epsilon_0=0.1$.*

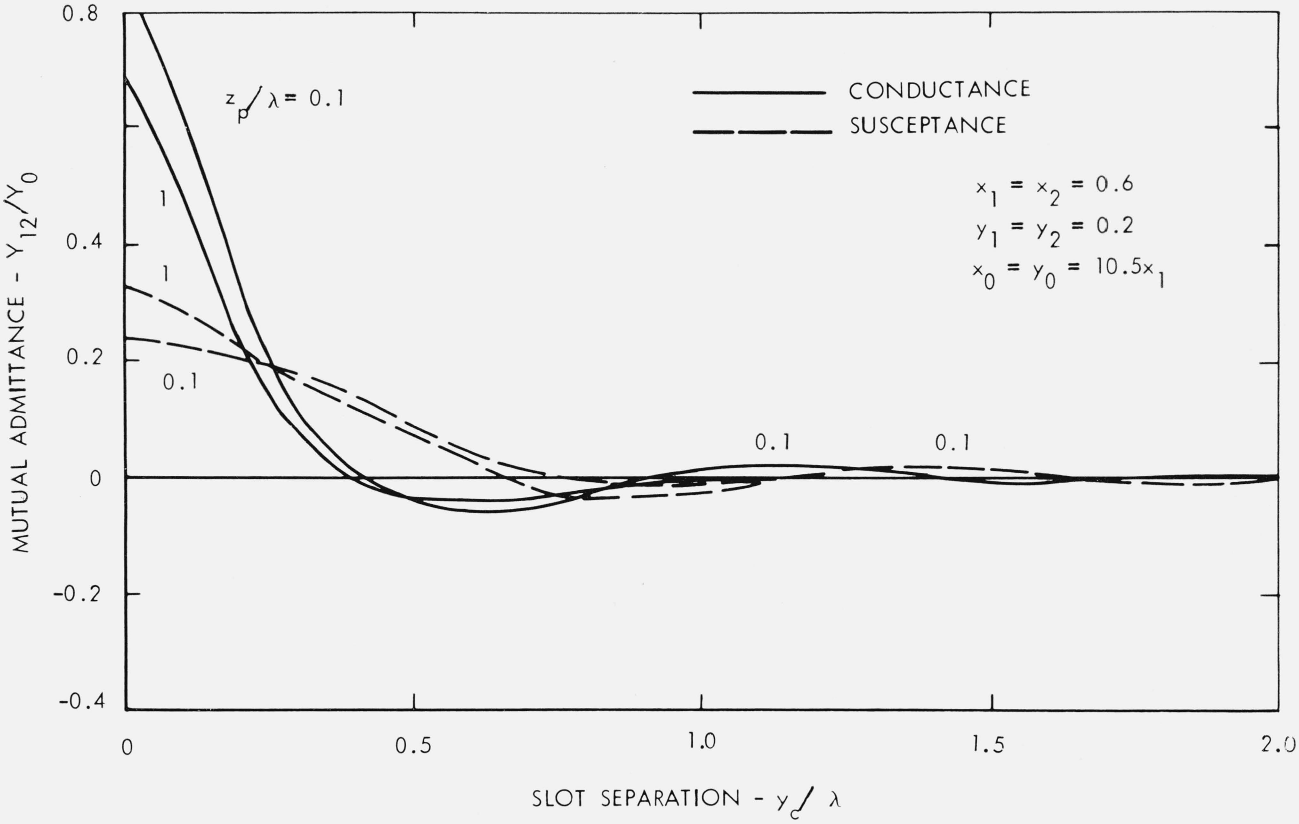

FIGURE 8. *Mutual admittance for a lossy plasma layer of* $\epsilon_p/\epsilon_0=0.5$, *Tan* $\delta_p=1$.

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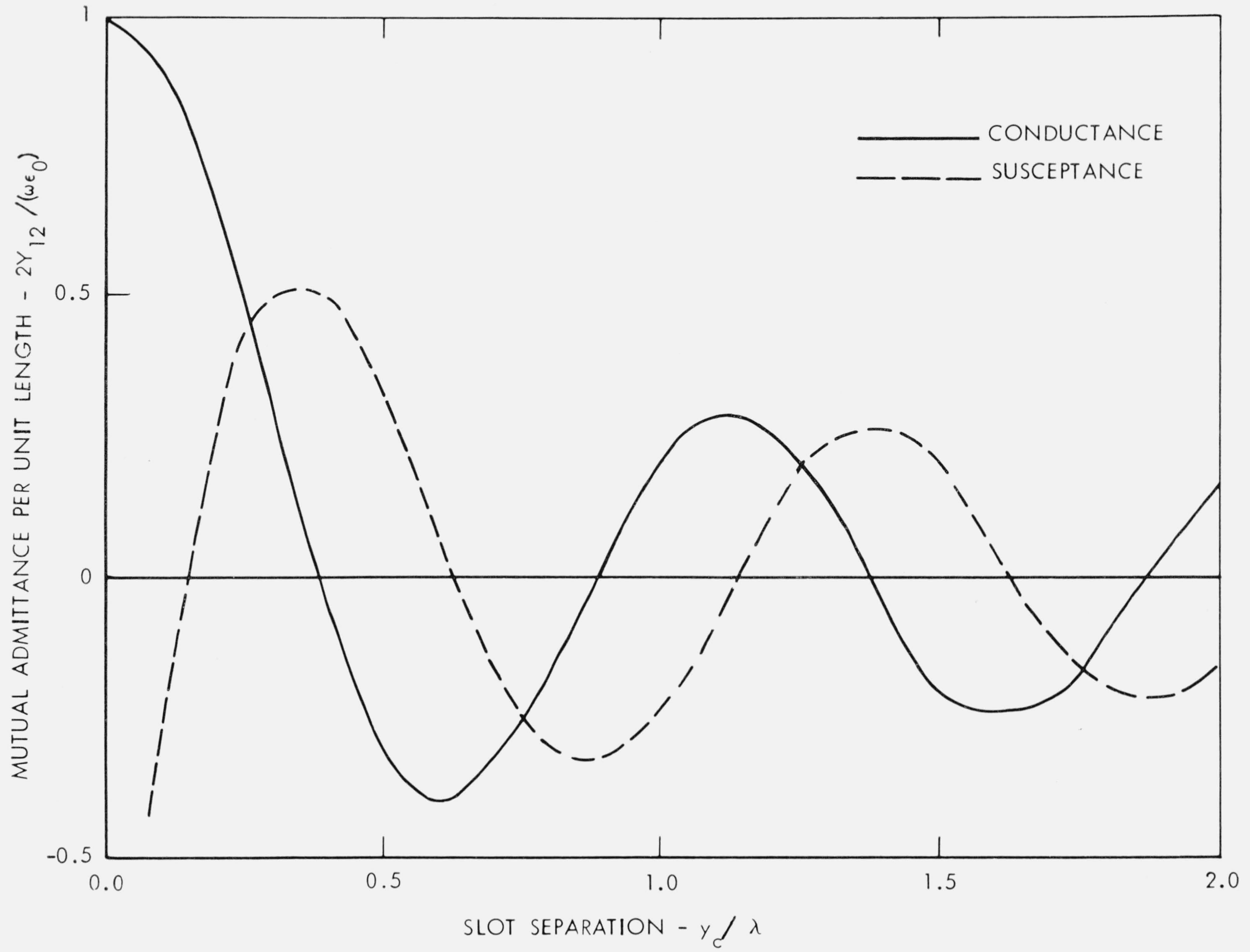

FIGURE 9. *Free space mutual admittance between two thin infinitely long slots.*

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